2014年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2012

学科試験 問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数 学

MATHEMATICS

注意 ☆試験時間は60分

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.

(2014)

MATHEMATICS

Nationality		No.	
Name	(Please print full name, family name)	undei	rlining

Marks

1 Answer all questions and write your answers in the boxes provided.

1) If
$$x = \frac{\sqrt{5}+2}{\sqrt{5}-2}$$
 and $y = \frac{\sqrt{5}-2}{\sqrt{5}+2}$. Find the value of $x^2 + y^2$.

$$x^2 + y^2 =$$

2) Solve the equation $x^3 + 6x^2 + 11x + 6 = 0$.

$$x =$$

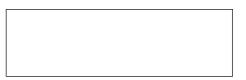
3) Solve the equation $\sin x - \cos x - \frac{1}{\sqrt{2}} = 0 \ (0 \le x < 2\pi).$

$$x =$$

4) Solve the equation $2\log_9(x+2) + \log_3 x = 1$.

$$x =$$

5	Solve	the	inea	uality	2^{1-x} —	2^{x+2}	<	7.
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6) Solve the inequality $\cos 2x + 9\sin x - 5 < 0 \ (0 \le x < 2\pi)$.



7) Find the coordinates of the point A' which is the symmetric point of A(3, 2) with respect to the line 2x + y - 12 = 0.

$$A'($$
 ,)

8) Two coins are tossed at the same time. What is the probability that both coins show heads under the condition that at least one coin shows heads.



9) The sequence $\{a_n\}$ satisfies the following recurrence formula. Express the general term a_n in terms of n.

$$a_1 = 1$$
, $a_{n+1} = a_n + n + 1$ $(n = 1, 2, 3, \cdots)$

$$a_n =$$

10) Find $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$.



11) Find the derivative of the function $y = x \log_e \sqrt{x}$.

$$y' =$$

12) Find the indefinite integral $\int xe^{-2x} dx$.



2 Let
$$I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$$
, $O=\begin{pmatrix}0&0\\0&0\end{pmatrix}$. Let $A=\begin{pmatrix}0&-1\\1&a\end{pmatrix}$ satisfying $A^2+A+I=O$.

Answer the following questions and write your answers in the boxes provided.

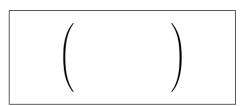
1) Evaluate a.

$$a =$$

2) Find A^3 .

$$A^3 =$$

3) Find $I + A + A^2 + \dots + A^{10}$.



- 3 Let $I = \int_0^{\pi} (a\cos x + b\sin x + 1)^2 dx$, where a and b are real numbers. Answer all questions and write your answers in the boxes provided.
 - 1) Calculate $\int_0^{\pi} \cos^2 x \, dx$.



2) To calculate the integral in I, express I as a function a and b.

$$I =$$

3) Find the minimum value of I and the value of a and b which minimize I.

$$\begin{aligned}
 &\min I = \\
 &a = \\
 &b =
 \end{aligned}$$